Foundations of Risk Management

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Teaching team

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Objectives

Risk Management is becoming increasingly important for financial institutions such as insurance companies, pension funds and banks. Various stakeholders (such as shareholders, policyholders, rating agencies, regulators) require them to perform a proper risk management of their business. This course introduces the student to risk measurement techniques and how these techniques can be applied to manage the risks faced by these financial institutions. After having followed this course, the student understands and is able to apply the most important concepts of expected utility utility theory and Yaari's dual theory of choice under risk.

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The student can explain how rational decision makers behave according to these theories. Furthermore, the student has a deep understanding and operational knowledge of the way how risk can be measured with so-called risk measures. The student knows the most important classes of risk measures (coherent risk measures, convex risk measures, distortion risk measures) and their properties (such as subadditivity). He/she also knows the properties of the most prominent members of these classes (VaR, TailVar, ESF). The student understands and can explain the relationships that exist between the theories of choice under risk and risk measurement.

Preliminaries

Given the quantitative nature of this course, a sound mathematical background and sufficient knowledge of probability theory and calculus is indispensable. The students should have knowledge about the following topics:

- calculus: Riemann and Lebesgue integrals and differentiation techniques;
- probability theory: distribution functions, expectations, normal and elliptical distributions.

Students must at least have mastered the topics, concepts and reasonings covered in the following international standard textbooks (or similar textbooks at undergraduate level):

- Ross, S. (2009) A First Course in Probability (8th edition), Prentice Hall.
- Barnett, R., Ziegler, M., Byleen, K. (2014). Calculus for Business, Economics, Life Sciences and Social Sciences (13th edition), Pearson.

Topics

Measuring risk is strongly related to measuring utility. Therefore, the course starts with an introduction to the classical expected utility theory. The famous St. Petersburg paradox is discussed and it is explained how it can be resolved with the help of utility theory. The concept of risk aversion is introduced. The relation between expected utility theory and insurance is investigated. The most important integral stochastic orders (stochastic dominance, stop-loss order, second degree stochastic dominance and convex order) are studied.

The Allais paradox within expected utility theory leads to the development of Yaari's dual theory of choice under risk. This dual theory is investigated, as well as its relationship with classical utility theory. The notion of risk aversion in this dual theory is investigated, as well as the relation between this theory and insurance. It is shown that the comonotonic dependence plays an important role in this theory. The relation between the most important integral stochastic orders and the dual theory of choice under risk is investigated.

When two or more risks are involved, the distribution of the aggregate risk depends on the marginal distributions and the dependence between the individual risks. The theory of copulas is introduced to show how dependence can be captured via a copula function. The most important copulas (Gaussian, t, Archimedean) are considered and their properties are studied. It is also shown how dependence can be captured in a single parameter, such as Pearson's correlation, Kendall's tau, etc.

The concept risk measure is introduced. Several of the properties that a risk measure may or may not have are considered. The classes of coherent risk measures and convex risk measures are defined. The most prominent risk measures (VaR and TailVar) are investigated. The relation between these prominent measures of risk and integral stochastic orders is studied.

Risk measurement of aggregate claims of a portfolio of risks is investigated. Properties of particular risk measure for the aggregate claims of such a portfolio are considered. Also convex order upper bounds for risk measures of such sums are studied. The famous subadditivity axiom of the class of coherent risk measures is investigated in detail. The relation between this axiom and diversification is explained. The consequences of a risk measure not being subadditivity are studied.

We consider the following topics:

- Topic 1: Utility theory:
  - decision making through expected utilities;
  - expected utility and insurance premiums;
  - mutual exclusivity;
  - exponential utility.
- Topic 2: Integral stochastic orders
  - classifying risks;
  - stochastic dominance;
  - stop loss premium and stop-loss order;
  - convex order;
  - Distributions and stop-loss transforms.
- Topic 3: Modeling, Measuring and comparing dependence
  - joint distribution functions;
  - multivariate Gaussian distribution;
  - copula functions
  - examples: Gaussian, t, Archimedean copulas;
  - dependence measures.
  - Multivariate Stochastic Orders

- Topic 4: Comonotonicity
  - a general framework for inverse cdf's;
  - comonotonic random vectors;
  - sums of comonotonic risks;
  - comonotonicity and elliptical distributions;
  - risk sharing and comonotonicity.
- Topic 5: The dual theory of choice under risk;
  - distorted probabilities and expectations: definition and properties;
  - Yaari's dual theory of choice under risk;
  - distorted expectations and insurance.
- Topic 6: Risk measures:
  - risk measures and solvency capital;
  - classes of risk measures: coherent and convex risk measures;
  - VaR and TVaR: definition and properties;
  - other risk measures.
- Topic 7: Subadditivity and aggregating risks:
  - stop-loss premium for comonotonic risks;
  - convex upper bounds for sums of risks;
  - diversification and subadditivity;
  - convex upper bounds and risk measures.
- Topic 8: Distortion risk measures
  - VaR and TVaR are distortion risk measures;
  - subadditivity of TVaR;
  - concave distortion risk measures.
- Topic 9: Workshop on aggregating risks
  - Monte Carlo simulation and normal distributions;
  - Monte Carlo simulation and Gaussian copulas;
  - Monte Carlo simulation and t-copulas;
  - Aggregating lognormal risks and risk measures;
  - **Note:** *The implementation is demonstrated using MatLab. In order to optimally benefit from this workshop, a laptop is required.*

- Topic 10: Application of comonotonicity: Aggregating lognormal risks
  - Convex upper and lower bounds;
  - aggregating independent risks;
  - aggregating comonotonic risks;
  - aggregating dependent lognormal risks;
  - applications.

Timetable

	Date	Content	Room
1	September 30, 9-11 AM	Topic 1	HOGC 02.28
2	September 30, 1-4 PM	Topic 2	HOG 01.85
3	October 1, 9-11 AM	Topic 3	PSI 2.51
4	October 21	Topic 4	Online Lecture
5	November 4	Exercises	Online Lecture
6	November 18	Topic 5	Online Lecture
7	November 25, 9-11 AM	Topic 6	HOGC 02.28
8	November 25, 1-4 PM	Topic 7	HOG 01.85
9	December 2, 9-11 AM	Topic 8	HOGC 02.28
10	December 9, 9-11 AM	Topic 9	Online Lecture
11	December 16, 9-11 AM	Topic 10	HOGC 02.28
12	December 16, 1-4 PM	Topic 10 (Continued)	HOG 01.85

Course material

- Lecture notes: Measuring and Managing Actuarial Risks. Jan Dhaene, Michel Denuit, Rob Kaas, Daniël Linders & Tim Verdonck.
- Presentations used during the teaching hours.
- Exercise sheets used during the exercise sessions.
- Additional notes and workings for the lectures and the workshops.
- Notice that all course material can be downloaded from Toledo and from the website: https://daniellinders.com/fqrm/.

Exam

An **open book written exam** is organized during the regular examination period. Only material that has been considered during the lessons has to be studied. The open book exam consists of a series of theoretical and/or numerical questions and exercises. The goal of the exam is to verify whether the student understands the concepts studied in the course, and also to verify whether he can apply what has been learned in specific actuarial risk management problems. The level of the exam questions is in accordance with the exercises that are available on toledo and / or have been solved during the exercise sessions.

Important: During the exam only a copy of the material that has been used during the classes and has been posted on Toledo (course notes, slides, exercises) can be consulted. **It is not allowed to add your own notes on this material.** Please bring your own pocket calculator.