

Foundations of Quantitative Risk Measurement

Chapter 6: Homework

Daniël Linders

November 28, 2019

1. Consider the following distortion function g :

$$g(q) = \min \left\{ \frac{q}{1-p}, 1 \right\}, \quad (0.1)$$

where $p \in (0, 1)$.

- (a) Show that the distorted expectation $\rho_g[X]$ can be expressed as follows:

$$\rho_g[X] = \text{TVaR}_p[X].$$

Remark: There exists a distortion function g such that the distorted expectation corresponds with the Tail Value-at-Risk. We say that the Tail Value-at-Risk is a *distortion risk measure*.

- (b) Consider the r.v. Y with cdf F_Y , which is defined as follows:

$$F_Y(x) = \begin{cases} 0, & x < F_X^{-1}(p); \\ \frac{F_X(x)-p}{1-p}, & x \geq F_X^{-1}(p). \end{cases}$$

Prove that

$$\rho_g[X] = \mathbb{E}[Y].$$

- (c) Assume that F_X is continuous and strictly increasing. Prove that

$$Y \stackrel{d}{=} X | X \geq F_X^{-1}(p).$$

Which of the two conditions, continuity or strictly increasingness, can be removed?